

Lorentz Transformation of Electromagnetic Systems and the 4/3 Problem

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Lorentz transformations of two macroscopic devices are discussed. In each case, the overall momentum flux into every static element of matter vanishes. It is shown that a Lorentz transformation of the energy-momentum 4-vector of each system agrees with special relativity. In particular, using the ordinary definition of 4-momentum of electromagnetic fields, it is proved by means of a particular form of Poincaré's stress tensor, that there is no 4/3 factor in the transformation of the entire momentum of a uniformly charged spherical shell.

1. Introduction

The problem of Lorentz transformation of energy-momentum of electromagnetic fields is still unsettled. An example of this point is the old claim that the 4-momentum of the transformed system of a uniformly charged spherical shell contains a 4/3 factor in a component of electromagnetic fields momentum. Differences of opinions and references to earlier works have been published several years ago [1, 2]. Further papers have followed this debate [3–5]. The purpose of the present work is to show that Lorentz transformations of ordinary definition of electromagnetic energy-momentum of two kinds of macroscopic systems agree completely with special relativity. A general theoretical law related to this point is discussed.

The systems analyzed here have the following common properties. All systems consist of macroscopic nonelementary devices. In a particular inertial frame, parts of the material components of each system are motionless and the entire system is stationary. The devices are constructed in such a way that for every infinitesimal element of a massive motionless constituent, momentum flux exchanged between it and the rest of the system vanishes. Besides the motionless massive components, the systems consist of electromagnetic fields and of massive particles, where the motion of the latter is confined to a limited volume. In all cases, the

systems are closed and exchange no energy-momentum with the rest of the world. Lorentz transformations of mechanical particles are well determined and they are used for deriving the main results of the present work. In particular, the overall momentum balance achieved by the introduction of appropriate mechanical constituents, plays a significant role in the clarification of the form of electromagnetic fields' momentum. The analysis uses extensively general properties of the energy-momentum tensor, and it is shown that the problems discussed are specific illustrations of them.

Expressions are written in units where the speed of light is $c=1$. Greek indices range from 0 to 3 and Latin ones take the values 1, 2, and 3. The spatial coordinates are denoted sometimes by the subscripts x , y , and z , respectively. The metric $g_{\mu\nu}$ is diagonal and its entries are $(1, -1, -1, -1)$. $F^{\mu\nu}$ denotes the anti-symmetric tensor of electromagnetic fields.

The structure of the paper is as follows. A problem of a parallel plates capacitor is discussed in the second section. In this case, a balance of momentum flux into every element of the plates is reached by means of special molecules which move perpendicular to the plates. The main results of this section are derived again in the third section where a charged spherical shell is analyzed. Here, a particular form of Poincaré's stress tensor [6, 7] serves as a balancing source of momentum current. The fourth section contains a discussion concerning underlying reasons for the results obtained in the second and the third sections. Concluding remarks are the contents of the last section.

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2. A Parallel Plates Capacitor

Consider a capacitor whose plates are parallel to the (y, z) plane. The size of its plates is much larger than the distance d between them. The plates are motionless in the laboratory frame Σ and their x -coordinates are 0 and d , respectively. Each plate is covered respectively with a uniform charge density $\pm \rho$. Lorentz velocity transformation in the x -direction is discussed. In the system described above, the momentum flux into each plate does not vanish. The final setup of the device is such that the plates and every element of them do not exchange momentum with other parts of the system. This goal is attained by a special kind of “molecules” that move parallel to the x -axis. The molecules do not interact with one another and their motion parallel to the x -axis is conserved. These molecules move between the capacitor’s plates and are reflected elastically by the latter. The molecules do not change the dielectric properties of the medium between the plates which is the same as that of the vacuum.

Using a 3-vector notation, one finds that, between the plates, the electrostatic field is

$$\mathbf{E} = 4\pi\rho(1, 0, 0), \quad (1)$$

whereas $E=0$ elsewhere. The symmetric energy-momentum tensor of electromagnetic fields is [8]

$$T_{(\Gamma)}^{\mu\nu} = \frac{1}{4\pi} \left(F^{\mu\alpha} F^{\beta\nu} g_{\alpha\beta} + \frac{1}{4} F^{\alpha\beta} F_{\alpha\beta} g^{\mu\nu} \right) \quad (2)$$

and its spatial part is Maxwell’s stress tensor

$$T_{ij} = \frac{1}{4\pi} \left[-E_i E_j - B_i B_j + \frac{1}{2} (E^2 + B^2) \delta_{ij} \right]. \quad (3)$$

Let us examine an infinitesimal volume $dV = S \cdot dx$ where S is a unit surface at the right hand plate of the capacitor and dV includes S . Using the null field outside the capacitor, one finds that the active face of dV is its inner one. It follows from (1) and (3) that the x -component of the momentum flux into dV

$$T_{xx} = -\frac{1}{8\pi} E^2. \quad (4)$$

This relation means that a net force pointing towards the other plate is exerted on dV . Obviously, the same conclusion is obtained from the Lorentz force exerted on charges at dV where the field used is associated with charges located outside dV .

Consider the energy-momentum contained in the volume V of a rectangular parallel pipe having unit bases, each of which lies on one of the capacitor’s plates. The height and volume of this rectangular parallel pipe is d . Let μ denote the mass density of the plates. Then, the 4-momentum of the two plates is

$$P_{(m)}^\mu = (2\mu, 0, 0, 0). \quad (5)$$

Let us turn to the fields 4-momentum. Evidently, the momentum density of the electrostatic field vanishes because $\mathbf{B}=0$. The energy of the fields is nonzero and the corresponding 4-momentum is

$$P_{(f)}^\mu = \frac{1}{8\pi} E^2 d(1, 0, 0, 0). \quad (6)$$

Let N be the number of molecules at the volume of the rectangular parallel pipe and N/d is their uniform density. The mass of every molecule is m . In the inertial frame Σ , the velocity of $N/2$ molecules is $\mathbf{v}_+ = (v, 0, 0)$ and the same number of molecules move in velocity $\mathbf{v}_- = (-v, 0, 0)$.

The 4-momentum of a molecule is

$$p_{\pm}^\mu = m \gamma_v (1, \pm v, 0, 0), \quad (7)$$

where $\gamma_v = (1 - v^2)^{-1/2}$. When a molecule hits the right hand plate, it transfers a momentum $2\gamma_v m v$ to the plate. Since the velocity of half of the molecules is \mathbf{v}_+ , one finds that the flux of mechanical momentum into the plates is

$$P = \frac{N}{d} \gamma_v m v^2. \quad (8)$$

The variables N , m , and v can be adjusted so that the electromagnetic momentum flux (4) is balanced. This requirement yields the following relation between the variables of the molecules and the electrostatic field

$$\frac{N}{d} \gamma_v m v^2 = \frac{1}{8\pi} E^2. \quad (9)$$

On the basis of (5)–(9), one finds that the overall 4-momentum of all physical constituents included in the volume of the rectangular parallel pipe is

$$P_{\text{tot}}^\mu = [2\mu + N m \gamma_v (1 + v^2)] (1, 0, 0, 0). \quad (10)$$

This discussion uses the time-average of the momentum exchanged between the molecules and the plates. This approximation is justified if, for example, one takes the limit of $N \rightarrow \infty$ and $m \rightarrow 0$ while the product $N m$ remains a constant.

Consider a Lorentz velocity transformation in the x -direction

$$L_v^\mu = \begin{pmatrix} \gamma & \beta & 0 & 0 \\ \beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (11)$$

where $\gamma = (1 - u^2)^{-1/2}$ and $\beta = u\gamma$. Transformation (11) takes us from the laboratory frame Σ to a frame Σ' where the plates are seen moving in velocity $\mathbf{u} = (u, 0, 0)$. The results of this transformation are as follows. The distance d between the plates and the volume V of the rectangular parallel pipe undergo a Lorentz contraction

$$d' = d/\gamma; \quad V' = V/\gamma. \quad (12)$$

The 4-momentum (5) of the motionless massive parts transforms as usual

$$p_m^\mu = 2\mu(\gamma, \beta, 0, 0). \quad (13)$$

A transformation of the 4-momentum (7) of a molecule yields

$$p_\pm^\mu = m\gamma\gamma_v(1 \pm uv, u \pm v, 0, 0), \quad (14)$$

The overall 4-momentum of the molecules is not obtained directly from (14) because, in Σ' , the number of molecules moving in velocity \mathbf{v}_+ is *not equal* to the number of molecules moving in velocity \mathbf{v}_- .

To see this point, let us look at the event of a molecule seen in Σ' hitting the right hand plate at $t' = 0$. This event is $x^\mu = (0, d/\gamma, 0, 0)$. Transforming back to Σ , one applies the inverse of (11) and finds that, in the laboratory frame, the same event takes the form $x^\mu = (-ud, d, 0, 0)$. This result means that, in Σ , the same particle is seen hitting the right hand plate at $t < 0$. Hence, at $t = 0$ it moves towards the left hand plate and its distance from the right hand plate is vud .

As pointed out above, in Σ one half of the molecules move to the right, the other half is seen moving to the left and their density is uniform. Hence, the number of molecules seen in Σ' approaching the right hand plate is $N(1 + uv)/2$ whereas the number of molecules moving towards the left hand plate is $N(1 - uv)/2$. This result is illustrated in Figure 1. Using these values and (14), one obtains the overall 4-momentum of the molecules seen in Σ' approaching the right hand plate

$$p_+^\mu = \frac{N}{2} m\gamma\gamma_v(1 + uv)(1 + uv, u + v, 0, 0). \quad (15)$$

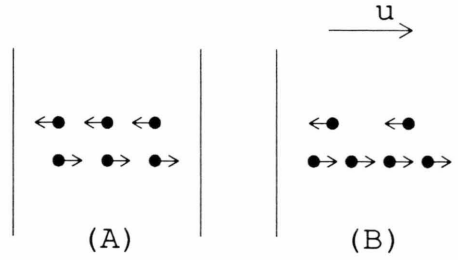


Fig. 1. A one dimensional gas is enclosed between capacitor's plates. (A) The situation as seen in the laboratory frame Σ . (B) The system, as seen in Σ' , where the plates move in velocity \mathbf{u} .

The corresponding expression for the molecules moving towards the left hand plate is

$$p_-^\mu = \frac{N}{2} m\gamma\gamma_v(1 - uv)(1 - uv, u - v, 0, 0). \quad (16)$$

The sum of (15) and (16) is the overall 4-momentum p_{mol}^μ of the molecules in Σ'

$$p_{\text{mol}}^\mu = N m\gamma\gamma_v(1 + u^2 v^2, u(1 + v^2), 0, 0). \quad (17)$$

In Σ' , the electrostatic field is the same as in Σ and its 4-momentum is

$$p_e^\mu = \frac{d}{8\pi} E^2/\gamma(1, 0, 0, 0) \quad (18)$$

which is just the 4-momentum of the capacitor's field in Σ , reduced by the factor γ^{-1} , due to the Lorentz contraction (12) of the volume of the rectangular parallel pipe.

Using (9) and the values of γ and β as defined after (11), one adds (13), (17), and (18) and obtains the total 4-momentum of the transformed physical constituents of the system

$$p_{\text{tot}}^\mu = [2\mu + N m\gamma_v(1 + v^2)](\gamma, \beta, 0, 0). \quad (19)$$

On the other hand, if one applies the Lorentz transformation (11) to the total 4-momentum (10) of the rectangular parallel pipe at Σ then he finds that the sum of the 4-momenta (19) of the transformed quantities equals the system's transformed 4-momentum. It follows that self-consistency of special relativity sustains.

The previous discussion relies on a detailed account of the molecules' motion and of their transformation. It is interesting to note that the same result is obtained from an analysis that considers the molecules as a continuous one dimensional macroscopic body char-

acterized by mass density and a one dimensional pressure. Reducing the energy-momentum tensor of a three dimensional gas (see [8], p. 85) to fit the one dimensional problem discussed here, one finds that, in the laboratory frame Σ , the energy-momentum tensor of the molecules discussed here is

$$T_{\text{gas}}^{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (20)$$

where

$$\varepsilon = N m \gamma_v / d \quad (21)$$

is the energy density of the molecules and the one dimensional pressure P is defined in (8). Applying the Lorentz transformation (11) to the tensor (20) one finds

$$T_{\text{gas}}^{\prime\mu\nu} = \begin{pmatrix} \gamma^2 \varepsilon + \beta^2 P & \gamma \beta (\varepsilon + P) & 0 & 0 \\ \gamma \beta (\varepsilon + P) & \beta^2 \varepsilon + \gamma^2 P & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (22)$$

This tensor is uniform over the volume of the rectangular parallel pipe. Integrating $T_{\text{gas}}^{\prime\mu 0}$ on the contracted volume (12) of the rectangular parallel pipe, one obtains

$$p_{\text{gas}}^{\prime\mu} = \frac{d}{\gamma} (\gamma^2 \varepsilon + \beta^2 P, \gamma \beta (\varepsilon + P), 0, 0). \quad (23)$$

A substitution of (8) and (21) into (23) yields (17). This conclusion shows that relativistic calculations of the discrete system of molecules moving in one dimension agree with the results obtained for a corresponding continuum. The conclusions found in this section are compatible with those of [9].

3. A Uniformly Charged Spherical Shell

In this section, a unit-radius spherical shell is discussed. The shell is covered uniformly with a charge density ϱ . It is motionless in the laboratory frame Σ and its center is at the origin. The electric field at an outer point next to the shell is radial and its strength is

$$E = 4\pi \varrho, \quad (24)$$

whereas, at inner parts of the shell, the field vanishes.

Let us consider an infinitesimal rectangular parallel pipe whose base is a surface element $d^2 S = dy dz$ of the shell at $\mathbf{r} = (1, 0, 0)$. The length of this rectangular parallel pipe is dx and $dx \ll dy$, $dx \ll dz$. Here, like in the case of the capacitor, the momentum flux into the rectangular parallel pipe, which is associated with the electric field, enters only from one side. However, in the present case, the active side is the outer face. Applying (3), one obtains (4), which takes here the form

$$T_{xx} = -\frac{1}{8\pi} E^2 = -2\pi \varrho^2. \quad (25)$$

This flux enters from the outer side, which means that a force pointing outwards is exerted on every surface element of the shell. Like in the case of the capacitor, the same conclusion can be obtained directly from the Lorentz force of the field associated with charges located outside the infinitesimal rectangular parallel pipe.

The analysis carried out here relies on relativistic properties of a uniform gas enclosed in a volume. Consider a unit cube whose axes are parallel to the x , y , and z -axes, respectively. The cube is full of gas made of N molecules which do not interact with one another. One third of the molecules move parallel to the x -axis, like the molecules discussed in the previous section and the rest move parallel to the y and z -axes, respectively.

The outcome of applying the Lorentz transformation (11) to the molecules moving parallel to the x -axis is discussed in Section 2. The 4-momentum of a molecule moving parallel to the y -axis is

$$p_{\pm}^{\mu} = m \gamma_v (1, 0, \pm v, 0). \quad (26)$$

Applying (11), one finds that, in Σ' , this 4-momentum takes the form

$$p_{\pm}^{\prime\mu} = m \gamma \gamma_v (1, u, \pm v/\gamma, 0). \quad (27)$$

In Σ' , the y -component of the velocity of $N/6$ molecules is positive and the same number have a negative y -component. Analogous properties hold for the molecules moving in Σ parallel to the z -axis.

In Σ , the total 4-momentum of the molecules enclosed inside the unit cube is

$$p_{\text{mol}}^{\mu} = N m \gamma_v (1, 0, 0, 0). \quad (28)$$

Using (17) and the foregoing properties of molecules moving parallel to the y and z -axes, one finds that, in

Σ' , the total 4-momentum of the molecules is

$$p_{\text{mol}}'^{\mu} = N m \gamma_v \left(1 + \frac{1}{3} u^2 v^2, u \left(1 + \frac{1}{3} v^2 \right), 0, 0 \right). \quad (29)$$

In Σ , the pressure is

$$P = \frac{N}{3} \gamma_v m v^2, \quad (30)$$

and the energy of the molecules is

$$\varepsilon = N \gamma_v m. \quad (31)$$

Hence, (29) can be put in the following form

$$p_{\text{mol}}'^{\mu} = \gamma (\varepsilon + P u^2, u(\varepsilon + P), 0, 0). \quad (32)$$

On the other hand, a system like this can be treated as a continuous macroscopic body (see Ref. 8, pp. 85–86). In the rest frame of a macroscopic body, the energy-momentum tensor of a gas is

$$T_{\text{gas}}^{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}, \quad (33)$$

where ε is the energy density and P is the pressure. The form of this tensor in a frame where the macroscopic body moves can be obtained from an application of the appropriate Lorentz transformation to (33) or, alternatively, by using a covariant representation of this tensor

$$T_{\text{gas}}^{\mu\nu} = (\varepsilon + P) u^{\mu} u^{\nu} - P g^{\mu\nu}, \quad (34)$$

where u^{μ} is the 4-velocity of the macroscopic body.

Like in the case of the capacitor, one adds to the system an appropriate component that is used for balancing the momentum flux into each surface element of the shell. This goal is achieved if the shell contains a particular “gas” whose self energy and pressure are negative. In this case, (25) shows that the total

Let M denote the mass of the spherical shell and $-\varepsilon$ is the energy density of the gas enclosed in it as seen in Σ . Hence, in Σ , the energy-momentum of the gas (33) takes the form

$$T_{\text{gas}}^{\mu\nu} = \begin{pmatrix} -\varepsilon & 0 & 0 & 0 \\ 0 & -2\pi q^2 & 0 & 0 \\ 0 & 0 & -2\pi q^2 & 0 \\ 0 & 0 & 0 & -2\pi q^2 \end{pmatrix}. \quad (36)$$

The energy of the electrostatic field is

$$W_e = \frac{1}{8\pi} \int E^2 d^3r = 8\pi^2 q^2. \quad (37)$$

It follows that the system's 4-momentum at the laboratory frame Σ is

$$p_{\text{tot}}^{\mu} = \left(M - \frac{4}{3} \pi \varepsilon + 8\pi^2 q^2 \right) (1, 0, 0, 0). \quad (38)$$

Let us apply the Lorentz transformation (11) to the quantities found in Σ . The electromagnetic fields 4-momentum is (see Appendix A)

$$p_e'^{\mu} = 8\pi^2 q^2 \left(\gamma \left(1 + \frac{1}{3} u^2 \right), \frac{4}{3} \beta, 0, 0 \right). \quad (39)$$

It is interesting to note that, in (39), neither the momentum nor the energy of the fields of this system transform like a 4-vector. It is proved below that this property is mandatory. The factor 4/3 in the momentum is debated for a long time (see [1, 2] and references therein).

The 4-momentum of the massive parts of the shell is as usual

$$p_{\text{shell}}'^{\mu} = M(\gamma, \beta, 0, 0). \quad (40)$$

The energy-momentum tensor of the gas at Σ' is obtained either from the application of (11) to (36) or from a utilization of (34) and (35). Using $\gamma^2 = \beta^2 + 1$, one finds

$$T_{\text{gas}}'^{\mu\nu} = \begin{pmatrix} -\varepsilon \gamma^2 - 2\pi q^2 \beta^2 & -(\varepsilon + 2\pi q^2) \gamma \beta & 0 & 0 \\ -(\varepsilon + 2\pi q^2) \gamma \beta & -\varepsilon \beta^2 - 2\pi q^2 \gamma^2 & 0 & 0 \\ 0 & 0 & -2\pi q^2 & 0 \\ 0 & 0 & 0 & -2\pi q^2 \end{pmatrix}. \quad (41)$$

momentum transmitted to a surface element of the shell vanishes if the pressure of the gas is

$$P = -2\pi q^2. \quad (35)$$

The integration of $T_{\text{gas}}'^{\mu 0}$ over the contracted volume of the spherical shell yields the 4-momentum of the gas in Σ' . Using the uniformity of the gas inside the shell,

one just multiplies $T_{\text{gas}}^{\prime\mu 0}$ by $4\pi/3\gamma$ and finds

$$p_{\text{gas}}^{\prime\mu} = -\frac{4\pi}{3}\gamma(\varepsilon + 2\pi\varrho^2 u^2, (\varepsilon + 2\pi\varrho^2)u, 0, 0), \quad (42)$$

where the relations $\gamma^2 - 1 = \beta^2 = \gamma^2 u^2$ are used. This expression agrees with (32).

The sum of (39), (40) and (42) is the overall 4-momentum of the physical constituents of the system, as seen in Σ'

$$p_{\text{tot}}^{\prime\mu} = \left(M - \frac{4\pi}{3}\varepsilon + 8\pi^2\varrho^2 \right) (\gamma, \beta, 0, 0). \quad (43)$$

Equation (43) and the result obtained from a direct application of the Lorentz transformation (11) to the system's total 4-momentum at Σ , (38), are identical. Here, like in the previous section, it is proved that special relativity is self-consistent if one considers Lorentz transformations of all physical elements of the system. The results of this section are in agreement with those of [3].

4. Discussion

The outcome of the foregoing analysis emerges from a fundamental property of the energy-momentum tensor. It is proved that if

$$T_{,\nu}^{\mu\nu} = 0 \quad (44)$$

then the integrals over the entire 3-dimensional space

$$p^\mu = \int_V T^{\mu 0} d^3r \quad (45)$$

is a conserved quantity (see [8], pp. 71–73, 77–80). Moreover, the integrals (45) are independent of the space-like hypersurface on which the integration is carried out. It follows that p^μ of (45) transforms like a 4-vector. This is the reason for the incorporation of the gaseous substances in the previous sections. As a result, the sum $T_e^{\mu\nu} + T_{\text{gas}}^{\mu\nu}$ does not transfer energy-momentum to other parts of the system, and the sufficient condition (44) is satisfied. On the basis of this general proof, the calculations of the second and the third sections can be considered as special examples illustrating the general theorem mentioned above.

This point applies directly to the case of the spherical shell where the integration of $T^{\mu 0}$ is carried out over the entire 3-dimensional space. On the other hand, in the second section the integration is carried out on the limited volume of the rectangular parallel pipe. However, it can be shown that the example pre-

sented in the second section is legitimate. To this end, let us examine special relativity in one time and one space coordinates. Here the region of integration is $(-\infty, \infty)$ and, in Σ , the electric field vanishes for $x < 0$ and for $x > d$. The example of the capacitor can be treated within this framework. Evidently, the proof of [8] holds also in the case of one spatial dimension. These points show that the discussion of the second section is a legitimate illustration of the general theorem.

Another point is the unphysical properties of the “gases” used, like the one dimensional gas of the second section and the negative sign of the energy and pressure of the gas of the third section. These properties do not affect the main objective of this work, which aims to analyze *mathematical* self-consistency of special relativity. This assignment can be taken within the realm of mathematics where thought experiments are analyzed and the impossibility of testing the results in a real experiment does not impair their significance.

5. Concluding Remarks

As pointed out in the introduction, every motionless massive element of a system analyzed here exchanges no energy-momentum with other parts of it. Constituents of the system are made of matter part whose properties are expressed in terms of energy-momentum 4-vector and of continuous elements, like fields and gas, whose properties are given by their energy-momentum tensor. Explicit calculations show that the 4-momentum of the entire system is a conserved quantity that transforms like a 4-vector. It is also shown that the same expressions are obtained if the molecules discussed here are considered as moving particles or as a continuous gaseous substance. These conclusions are in accordance with general physical requirements.

The effect of the electromagnetic fields on charged matter is compensated here by means of special gaseous substances. The laws of transformation of mechanical constituents of the system, namely, the matter on which charge is located and the moving molecules, is beyond any doubt. This point is used in the second and the third sections where relativistic properties of the motion of microscopic particles are calculated and is derived also from the representation of the system as a continuous macroscopic body.

Since the entire 4-momentum of every system discussed here is a sum of a mechanical part and an

electromagnetic part, then an acceptance of the form of the transformed mechanical components means that the properties found above for Lorentz transformations of energy-momentum of electro-magnetic fields are mandatory: an alteration of these laws, if yields different values of the fields 4-momentum $\int T^{\mu 0} d^3 r$, destroys 4-momentum conservation of the *entire* system. This argument supports the conventional definition of electromagnetic momentum which takes, in every frame, the form $\frac{1}{4\pi} \int \mathbf{E} \times \mathbf{B} d^3 r$ [8], whereas definitions whose integrals disagree with it are wrong.

Another aspect of the results obtained above is that p^μ of electromagnetic fields of a capacitor and of a charged spherical shell does not transform like a 4-vector. The same is true for a macroscopic mechanical element whose pressure does not vanish. These conclusions should not be surprising because, in the example discussed here, the energy-momentum tensor of fields as well as that of the gaseous substances do not satisfy requirement (44). The devices are built so that only the sum of these tensors is compatible with (44).

As mentioned in the introduction, the 4/3 factor in the momentum of transformed fields has been discussed in the literature for a long time. It is shown here that this is just an example of electromagnetic fields interacting with charges and for which requirement (44) does not hold. Here the 4/3 factor emerges from the spherical symmetry of the problem. In the example of the capacitor presented in the second section there is no 4/3 factor. Here, although the device is seen moving in Σ' , the momentum of the transformed fields *vanishes* and their energy *decreases* by the factor γ^{-1} . As shown in (19), these features are in complete agreement with special relativity.

As pointed out in the introduction, the devices considered here are macroscopic bodies. On the other hand, in the case of an elementary classical point charge, one cannot use additional components whose pressure balances the system because such components are incompatible with the elementary nature of the particle. It follows that for an elementary point

charge, the form (2) of the fields energy-momentum tensor cannot be used. The problem of the energy-momentum of the fields of a system made of elementary classical point charges will be discussed elsewhere [10].

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Appendix A

The calculation follows a method described in the appendix of [1]. The integration is carried out on the 3-dimensional space of Σ' where values of the fields are taken at the same time t' . Performing a change of variables from (t', x', y', z') to (t', x, y, z) , one can take advantage of the spherical symmetry of the problem at Σ . The matrix of this transformation is

$$R = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (46)$$

and γ is the corresponding Jacobian.

Let us evaluate the fields energy. At a point \mathbf{r} the energy of the transformed fields is

$$\begin{aligned} E'^2 + B'^2 &= E_x^2 + \gamma^2(E_y^2 + E_z^2) + u^2 \gamma^2(E_y^2 + E_z^2) \\ &= \gamma^2[(1 - u^2)E_x^2 + (1 + u^2)(E_y^2 + E_z^2)], \end{aligned} \quad (47)$$

where $\gamma^2(1 - u^2) = 1$ is used. Due to the spherical symmetry, it is evident that, after the integration is performed, each of the integrands E_x^2 , E_y^2 , E_z^2 yields the same value as $\frac{1}{3}E^2$. Using the foregoing arguments, one finds that the required fields energy in Σ' is

$$\begin{aligned} \frac{1}{8\pi} \int (E'^2 + B'^2) d^3 r' &= \frac{1}{8\pi} \gamma \int E^2 \left(1 + \frac{1}{3}u^2\right) d^3 r \\ &= \gamma \left(1 + \frac{1}{3}u^2\right) 8\pi^2 \varrho^2, \end{aligned} \quad (48)$$

where (37) is used. This is the 0-component of (39).

The calculation of the 1-component of (39) can be carried out analogously and is presented in [1].

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